Exploration in the Boson Sector of the Glashow-Salam-Weinberg model

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11 July 2012

Relations between Z, W, Higgs masses and α are proposed.

Inputs

1. Fine Structure Constant

$$\alpha \equiv \frac{e^2}{4\pi} \tag{1}$$

$$\alpha^{-1} = 137.035\ 999\ 037\ (91)[1] \tag{2}$$

$$\alpha^{-1} = 137.035\ 999\ 166\ (34)[2] \tag{3}$$

2. Z and W Masses [3]

$$m_Z = 91.1876\ (21)\text{GeV} \tag{4}$$

$$m_W = 80.385\ (15)\text{GeV} \tag{5}$$

3. Fermi Constant[3]

$$G_F = 1.166\ 378\ 7\ (6) \times 10^{-5}\ \text{GeV}^{-2} \tag{6}$$

4. Scalar Boson Mass (BEH, Brout-Englert-Higgs)

$$m_H = 125.3\ (6)\ \text{GeV}[4] \tag{7}$$

$$m_H \approx 126.5\ \text{GeV}[5] \tag{8}$$

5. Top Quark Mass

$$m_t = 173.2\ (9)\ \text{GeV}[6] \tag{9}$$

$$m_t = 173.3\ (9)\ \text{GeV}[7] \tag{10}$$

Empirical Relations

1. Defining

$$\cos\theta_W = \frac{m_W}{m_Z} \tag{11}$$

we consider the following empirical relation [7]

$$1 - \tan^2\left(\frac{\pi}{4} - \theta_W\right) = 3e. \tag{12}$$

From Eqs (1) or (2) and (12), we find

$$\tan\left(\frac{\pi}{4} - \theta_W\right) = 0.302545265 \tag{13}$$

to be compared with

$$e = \sqrt{4\pi\alpha} = 0.302822121. \tag{14}$$

From Eqs (4), (11) and (13), we get

$$m_W = 80.3887\ (19)\ \text{GeV} \tag{15}$$

to be compared with Eq. (5).

2. Let us consider the relation [7]

$$1 - e^2(1-\delta) = 3e. \tag{16}$$

Comparison between Eqs (12) and (16) imply [7]

$$\tan^2\left(\frac{\pi}{4} - \theta_W\right) = e^2(1-\delta). \tag{17}$$

Let us solve Eq. (16):

$$\frac{1}{\pi} = \alpha\left[3 + \sqrt{13 - 4\delta}\right]^2. \tag{18}$$

By inspection, it is easy to see that [7]

$$\delta = \frac{\alpha}{4}\left(1 + \frac{\alpha}{4} + x\left(\frac{\alpha}{4}\right)^2\right). \tag{19}$$

If

$$\begin{aligned}
x &= 1 & \alpha^{-1} &= 137.035\,999\,004\,289\,209 \\
& \tfrac{1}{2} & & 137.035\,999\,074\,212\,585 \\
& \tfrac{1}{4} & & 137.035\,999\,109\,174\,273 \\
& 0 & & 137.035\,999\,144\,135\,961 \\
& -\tfrac{1}{4} & & 137.035\,999\,179\,097\,648 \\
& -\tfrac{1}{2} & & 137.035\,999\,214\,059\,336
\end{aligned}$$

to be compared with Eqs (2) or (3).

3. Mass Scales

$$\tan\left(\frac{\pi}{4} - \theta_W\right) = \frac{m_W - m_B}{m_W + m_B} = e(1-\delta)^{\frac{1}{2}}[7] \tag{20}$$

with $m_B = \sqrt{m_Z^2 - m_W^2}$.

If we assume

$$m_Z = \frac{e}{\sin\theta_W \cos\theta_W}\; m_H \tag{21}$$

from Eqs (2) or (3), (4) and (13) we get

$$m_H = 125.31 \text{ GeV}$$

to be compared with Eqs (7) or (8).

4. $\alpha, G_F, \cos\theta_W, m_t$ and ρ parameter [3]

$$\rho = \frac{3G_F}{8\sqrt{2}\pi^2}\left\{m_t^2 + m_b^2 - \frac{4m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\frac{m_t}{m_b}\right\}. \tag{22}$$

We observe that we find

$$\rho = \frac{\alpha}{\cos^2\theta_W} = 0.009389594 \tag{23}$$

if $m_t = 174.0$ GeV to be compared with Eqs (9) or (10).

Observe that the BEH field vacuum expectation value [3]

$$v_F = \left(\frac{1}{\sqrt{2}G_F}\right)^{\frac{1}{2}} = 246.220 \text{ GeV} \tag{24}$$

and $m_t \approx \dfrac{v_F}{\sqrt{2}} = 174.1$ GeV.

Appendix

Useful relations [7]

From Eqs (12) and (17), we obtain

$$\bullet \quad \frac{e}{\sin\theta_W \cos\theta_W} = \frac{4}{3}\frac{1}{(\cos\theta_W + \sin\theta_W)^2} = \frac{2}{3}\frac{1}{\cos^2(\frac{\pi}{4} - \theta_W)} = \frac{4}{3} - 2e \qquad (25)$$

$$\bullet \quad \tan^2 2\theta_W = \frac{9}{4}\,\frac{1}{1-\delta} \qquad (26)$$

$$\bullet \quad \cos^2\theta_W = \frac{1}{2}\left[1 + \sqrt{\frac{4}{13}}\sqrt{\frac{1-\delta}{1-\frac{4}{13}\delta}}\right]. \qquad (27)$$

It is worth to mention

$$m_W + m_B = \sqrt{2}\cos\Big(\frac{\pi}{4} - \theta_W\Big) m_Z = \frac{v}{2} \qquad (28)$$

with

$$v = (1+\epsilon) v_F \qquad (29)$$

and $\epsilon \approx 0.00266$.

Using Eqs (12), (20) and (28), we get [7]

$$m_W = \frac{1 + \tan\Big(\frac{\pi}{4} - \theta_W\Big)}{2}\,\frac{v}{2} = \frac{3e}{2\Big(1 - \tan\Big(\frac{\pi}{4} - \theta_W\Big)\Big)}\,\frac{v}{2} \qquad (30)$$

References

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