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New track in the standard model ?

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For 10 years, since the opening of the LHC, no fundamental particles beyond the standard model $SU(3) \times SU(2) \times U(1)$ have been discovered. In this situation, it seems desirable to us to follow E. Schrödinger's advice: "The task is, not so much to see what no one has yet seen, but to think what nobody has yet thought, about that which everybody sees." It is with this in mind that we will try to identify some "hidden" relationships between the masses of the heavy particles in the standard model, namely W, Z, H and t.

1. Data

Let's start by collecting the necessary data to base our explanation.

- W mass [1]:

$$m_W = 80.380 \pm 0.013 \text{ GeV (experimental data)} \tag{1a}$$

$$m_W = 80.360 \pm 0.006 \text{ GeV (global fit of standard model)} \tag{1b}$$

- Z mass [1]:

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV} \tag{2}$$

- H mass [1]:

$$m_H = 125.10 \pm 0.14 \text{ GeV} \tag{3}$$

- t mass:

$$m_t^{\text{pole}} = 172.90 \pm 0.47 \text{ GeV} \quad [1] \tag{4a}$$

$$m_t^{\text{pole}} = 173.1 \pm 0.9 \text{ GeV (from cross-section measurement)} \quad [2] \tag{4b}$$

- Higgs vacuum expectation value, v:

$$v = 246.21965 \pm 0.00006 \text{ GeV} \quad [1,2] \tag{5}$$

where $v = 1/(2^{1/2} G_F)^{1/2}$ and G_F is the Fermi coupling constant

- fine-structure constant, a [$a = e^2/4\pi$]

$$a^{-1} = 137.035\ 999\ 139\ (31) \quad \text{(CODATA 2014)} \quad [2] \tag{6a}$$

$$a^{-1} = 137.035\ 999\ 1491\ (331) \quad [3] \tag{6b}$$

$$a^{-1} = 137.935\ 999\ 046\ (27) \quad [4] \tag{6c}$$

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2. m_Z, m_W, v, e, $\tan(\pi/4 - \vartheta_W)$

From Equations (1a) and (1b), we can estimate the mass of W is around 80.365 GeV. We have such a result when

$$m_W - m_B = ev/2 \quad [5] \tag{7}$$

with

$$m_B = (m_Z^2 - m_W^2)^{1/2}.$$

Eq. (7) can be rewritten as follows

$$m_Z = [\cos\vartheta_W - \sin\vartheta_W]^{-1} \cdot ev/2 = 1/2^{1/2} \cdot [1 + \tan^2(\pi/4 - \vartheta_W)]^{1/2} \cdot e/\tan(\pi/4 - \vartheta_W) \cdot v/2 \tag{8}$$

with

$$\cos\vartheta_W = m_W/m_Z.$$

From Eq.(8), using Eq.(2) together with Eqs. (5) and (6a-c), we get

$$m_W = 80.3664 \pm 0.0016 \text{ GeV} \tag{9}$$

$$\tan(\pi/4 - \vartheta_W) = 0.301982 \mp 0.000008 \tag{10}$$

$$e/\tan(\pi/4 - \vartheta_W) = 1.002783 \pm 0.000025 \tag{11}$$

$$[1 - \tan^2(\pi/4 - \vartheta_W)]/3e = 1.000375 \pm 0.000005. \tag{12}$$

A possible common origin for e and $\tan(\pi/4 - \vartheta_W)$ cannot be excluded.

Observing that

$$[1 - e^2]/3e = 0.999815514 \tag{13}$$

we get

$$[1 - \tan^2(\pi/4 - \vartheta_W)]/3e + 2[1 - e^2]/3e = 3.000006 \pm 0.000005. \tag{14}$$

It is tentalizing to consider case where

$$[1 - \tan^2(\pi/4 - \vartheta_W)]/3e + 2[1 - e^2]/3e = 3 \tag{15}$$

Observing [6] that

$$[1 - e^2]/3e = 1 - (1/3)e\delta = 1 - 0.000184486 \tag{16}$$

with

$$\delta = a/4[1 + a/4 + x(a/4)^2], \tag{17}$$

[x is around 0 if a is given by Eq. (6a-b) and around 0.75 if a is given by Eq. (6c)], we deduce from Eq. (15) that

$$[1 - \tan^2(\pi/4 - \vartheta_W)]/3e = 1 + (2/3)e\delta = 1 + 0.000368972 \tag{18}$$

$$\tan^2(\pi/4 - \vartheta_W) = e^2(1 - 3\delta) = (0.301990793)^2 \tag{19}$$

Then, from Eq.(8), we get

$$m_Z = 91.1850 \text{ GeV} \tag{20}$$

and

$$m_W = 80.3646 \text{ GeV} \tag{21}$$

3. m_H, m_Z, v, e, $\tan(\pi/4 - \vartheta_W)$

From the analysisis of the data, we propose the following relation between m_H and m_Z:

$$\begin{aligned} m_Z &= e/(\sin\vartheta_W \cos\vartheta_W)\cdot e/\tan(\pi/4 - \vartheta_W)\cdot m_H \\ &= 2e\cdot[1 + \tan^2(\pi/4 - \vartheta_W)]/[1 - \tan^2(\pi/4 - \vartheta_W)]\cdot e/\tan(\pi/4 - \vartheta_W)\cdot m_H \,. \end{aligned} \tag{22}$$

From Eqs (8) and (22), we get

$$\begin{aligned} m_H &= (\sin\vartheta_W \cos\vartheta_W)/(\sin\vartheta_W + \cos\vartheta_W).\ v/2e \\ &= [1 - \tan^2(\pi/4 - \vartheta_W)]/[8(1 + \tan^2(\pi/4 - \vartheta_W))]^{1/2}\cdot v/2e \end{aligned} \tag{23}$$

With Eqs. (2), (5), (6a-c) and (10), we find

$$m_H = 125.0492 \pm 0.0009 \text{ GeV} \tag{24}$$

With Eqs.(5), (6a-c) and (19), we find

$$\mathrm{m_H} = 125.0481 \text{ GeV} \tag{25}$$

to be compared with Eq.(3)

Observe that

$$m_H \cdot m_Z = (\tan 2\vartheta_W)/2\cdot(v/2)^2 \tag{26}$$

It is worthwhile to go back to the known formula

$$m_Z = (\sin\vartheta_W \cos\vartheta_W)^{-1} \cdot (1 - \Delta r)^{-1/2} \cdot ev/2 \tag{27}$$

where Δr includes the radiative corrections relating alpha, G_F, m_W and mZ.

See Eq. (10.10) in Ref. [7].

Comparing Eq.(8) and (27), we find

$$(1 - \Delta r)^{1/}2 = 1/\sin\vartheta_W - 1/\cos\vartheta_W$$

where all radiative corrections are encapsuled.

4. m_t, m_H, m_Z, m_W and v

In the standard model, the masses of all elementary particles (H, Z, W, quarks, leptons) are obtained using the Brout-Englert-Higgs mechanism and are proportional to the Higgs vacuum expectation value, v. The experimental data suggest that v is reciprocally constituted by the masses of the elementary particles as follows [8]

$$m_H^2 + m_Z^2 + m_W^2 + m_t^2 + m_b^2 + m_c^2 + m_\tau^2 + \ldots = v^2$$

provided that $m_t = 173.8$ GeV. If m_t is smaller than 173.8 GeV, then there is room for Higgs boson to couple to dark particles, for example in the scalar singlet dark matter model [9].

Conclusion

It is not excluded that those obvious relations between particle masses in the standard model reflect deep connections.

After all, 150 years ago, when Dmitri Mendeleev proposed his first version of the Table of the Elements, he was in a similar situation!

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