Is the fine structure constant α arbitrary or fixed?

Jean Pestieau[1]

18 December 2008

Abstract

A simple relation relating the fine coupling constant and π implies
$\alpha^{-1} = 137.035\ 999\ 083\ 843.$

The most precise determination of α, the fine structure constant, is given in reference [1]:

$$\alpha^{-1} = 137.035\ 999\ 084\ (51) \tag{1}$$

or

$$e = \sqrt{4\pi\alpha} = 0.302\ 822\ 120\ 87\ (6) \tag{2}$$

We are going to indicate a remarkable relation between e (or α) and π. It is not an approximate relation as

$$\frac{3}{e\pi^2} = 1.0038 \tag{3}$$

or

$$\frac{1}{e\pi\cosh(\frac{1}{\pi})} = 1.000052302 \tag{4}$$

but it is a motivated and very precise one.

Firstly, we define e_0 and e_1, two parameters smaller and bigger than e respectively:

$$e_0 = \frac{\sqrt{13}-3}{2} = 0.302\ 775\ 637\ 731\ 995, \tag{5}$$

solution of equation

$$\frac{1}{e_0} - e_0 = 3 \tag{6}$$

introduced in references [2] et [3]

and

$$e_1 = \frac{2\pi}{2\pi^2+1} = 0.302\ 961\ 668\ 745\ 990. \tag{7}$$

[1]Institute of Theoretical Physics, Catholic University of Louvain, Belgium – jean.pestieau@uclouvain.be

There is a simple relation relating e, e_0 and e_1:

$$\left(\frac{1}{e}-\frac{3}{4e_0}-\frac{1}{4e_1}\right)+\sqrt{2}\left(e-\frac{3}{4}e_0-\frac{1}{4}e_1\right)=0. \tag{8}$$

Solution of eq. (8) is

$$e = 0.302\ 822\ 120\ 871\ 926 \tag{9}$$

or

$$\alpha^{-1} = 137.035\ 999\ 083\ 843, \tag{10}$$

in perfect agreement with eqs (1) et (2).

Let us examine eq. (8) more closely. It can be obtained from two equations depending of a parameter f in the following way:

$$e = \frac{3}{4}e_0+\frac{1}{4}e_1+f \tag{11}$$

$$\frac{1}{e} = \frac{3}{4e_0}+\frac{1}{4e_1}-\sqrt{2}f. \tag{12}$$

It is easy to compute f from eqs (11) and (12):

$$\sqrt{2}f^2-f\left[\left(\frac{3}{4e_0}+\frac{1}{4e_1}\right)-\sqrt{2}\left(\frac{3}{4}e_0+\frac{1}{4}e_1\right)\right]+\left[1-\left(\frac{3}{4e_0}+\frac{1}{4e_1}\right)\left(\frac{3}{4}e_0+\frac{1}{4}e_1\right)\right]=0. \tag{13}$$

The f value obtained from eq. (13) is

$$f = -0.000\ 000\ 024\ 613\ 569 \tag{14}$$

using eqs (5) and (7).

Substracting eq. (11) from eq. (12), we get, using eq. (6),

$$\frac{1}{e}-e = 3+\frac{1}{4}\left(\frac{1}{e_1}-e_1-3\right)-(1+\sqrt{2})f. \tag{15}$$

Eq. (15) is equivalent to eq. (8) as far as e determination is concerned. We see that eq. (6) is an approximate form of eq. (15) because, using eqs (7) and (14), we get from eq. (15) that

$$\frac{1}{e}-e = 2.999\ 446\ 541\ 406\ 331. \tag{16}$$

The central result of this note is e (or α) value given in eq. (9) (or in eq. (10)).

A way for motivating this note is to remember that experimentaly [4]

$$\cos\theta_W = \frac{m_W}{m_Z} = 0.8817 \pm 0.0003. \tag{17}$$

where m_W and m_Z are W and Z masses respectively.

From that we deduce

$$\tan\left(\frac{\pi}{4} - \theta_W\right) = 0.3028 \pm 0.0007 \tag{18}$$

$$2\tan 2\theta_W = 2.9993 \pm 0.0083. \tag{19}$$

As done in references [2] and [3], it is tempting to identify $\tan\left(\frac{\pi}{4} - \theta_W\right)$ and e:

$$\tan\left(\frac{\pi}{4} - \theta_W\right) = e = 0.302822. \tag{20}$$

The trigonometrical relation

$$\frac{1}{\tan(\frac{\pi}{4} - \theta_W)} - \tan\left(\frac{\pi}{4} - \theta_W\right) = 2\tan 2\theta_W \tag{21}$$

can be identified to eq. (16) using eqs (13) and (20).

It is not impossible that string inspired multiverse hypothesis is no more than a metaphysical bubble ready to burst at any time. If this occurs, it could help the entire community of theoretical physicists to go back to the physics of reality.
Let us conclude with a Feynmann's citation [5]:

> There is a most profound and beautiful question associated with the observed coupling constant, e—the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to -0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 wilh about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.)
>
> Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out—without putting it in secretly!

I thank Patrick Pestieau for many discussions and his calculatory help.

References

[1] T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, Phys. Rev. *D77*: 053012 (2008) ; arXiv: 0712.2607 (hep-ph)

[2] G. Lopez Castro, J. Pestieau, arXiv: 9804272 (hep-ph)

[3] G. Lopez Castro, J. Pestieau, Mod. Phys. Lett. *A22*, 2929 (2007) ; arXiv: 0609131 (hep-ph)

[4] C. Amsler *et al.*, *2008 Review of Particle Physics*, Phys. Lett. *B667*, 1 (2008)

[5] R.P. Feynman, QED–The strange theory of light and matter, Princeton University Press, 1985, p. 129