Variations on the Gauge Sector of the Electroweak Model

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Abstract

Starting from a 40 year old proposal, new relations between alpha, the fine structure constant, Z and W masses are proposed.

I. Forty years ago[(1)], it has been proposed the following determination of Z and W masses

$$\bar{m}_Z = \frac{A_0}{\sin\bar{\theta}\cos\bar{\theta}} \tag{1}$$

$$\bar{m}_W = \frac{A_0}{\sin\bar{\theta}} \tag{2}$$

with[(2)]

$$A_0 = \left(\frac{\pi\alpha}{\sqrt{2}G_F}\right)^{1/2} = 37.28057(8) \text{ GeV} \tag{3}$$

$$\sin\bar{\theta} = \sqrt{\frac{3}{14}}. \tag{4}$$

(The weak angle $\bar{\theta}$ in Eqs (1) and (2) is the complementary angle of θ defined in Ref (1) : $\theta + \bar{\theta} = \frac{\pi}{2}$).
Then

$$\bar{m}_Z = 90.85560(19) \text{ GeV} \tag{5}$$

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$$\bar{m}_W = 80.53524(17)\ \mathrm{GeV} \tag{6}$$

to be compared with their experimental values[2]

$$m_Z = 91.1876(21)\ \mathrm{GeV} \tag{7}$$

$$m_W = 80.398(25)\ \mathrm{GeV} \tag{8}$$

Let us present variations on Eqs (1) and (2).

II. It is amusing to consider the following simple parametrizations :

A.

$$m_Z = \bar{m}_Z(1 + \frac{\alpha}{2}) = 91.18750(19)\ \mathrm{GeV} \tag{9}$$

$$m_W = \bar{m}_W(1 + \frac{\alpha}{2})^{-1/2} = 80.38871(17)\ \mathrm{GeV} \tag{10}$$

B.

$$m_Z = \bar{m}_Z \left(\frac{\cos\bar{\theta}}{\cos\theta_W}\right)^{2/3} = 91.18757(19)\ \mathrm{GeV} \tag{11}$$

$$m_W = \bar{m}_W \left(\frac{\cos\theta_W}{\cos\bar{\theta}}\right)^{1/3} = 80.38868(17)\ \mathrm{GeV} \tag{12}$$

with[3]

$$\alpha = \frac{e^2}{4\pi} = [137.035999084(51)]^{-1} \tag{13}$$

$$\cos\theta_W \equiv \frac{m_W}{m_Z}. \tag{14}$$

We used value of $\cos\theta_W$ obtained from the empirical relation[4]

$$1 - \tan^2(\frac{\pi}{4} - \theta_W) = 3e. \tag{15}$$

Note that

$$1 - \tan^2(\frac{\pi}{4} - \theta_W) = \frac{4\sin\theta_W\cos\theta_W}{(\sin\theta_W + \cos\theta_W)^2}. \tag{16}$$

III. To make contact with a well known parametrization[2]

$$m_Z = \frac{A_0}{\sin\theta_W\cos\theta_W}\ \frac{1}{(1-\Delta r)^{1/2}} \tag{17}$$

we write Eq. (11) as

$$m_Z = \frac{A_0}{\sin\theta_W\cos\theta_W}\left(\frac{\sin\theta_W}{\sin\bar{\theta}}\right)\left(\frac{\cos\theta_W}{\cos\bar{\theta}}\right)^{1/3}. \tag{18}$$

Then

$$\frac{1}{(1-\Delta r)^{1/2}} = \left(\frac{\sin\theta_W}{\sin\bar{\theta}}\right)\left(\frac{\cos\theta_W}{\cos\bar{\theta}}\right)^{1/3} \tag{19}$$

in the current context.

IV. It is interesting to note the following empirical formula[(4)] :

$$\begin{aligned} m_Z &= \frac{1}{\sin\theta_W+\cos\theta_W}\left(\frac{\cos\bar{\theta}}{\cos\theta_W}\right)^{23/48}\frac{v_F}{2} && (20)\\ &= \frac{A_0}{\sin\theta_W\cos\theta_W}\frac{3}{4}(\sin\theta_W+\cos\theta_W)\left(\frac{\cos\bar{\theta}}{\cos\theta_W}\right)^{23/48} && (21)\\ &= 91.18756(19)\ \text{GeV} && (22)\end{aligned}$$

where we have used

$$A_0 = \frac{e\ v_F}{2} \tag{23}$$

and Eqs (15-16).

V. About e and α

With α and e given in Eqs (13), we satisfy the following Equation[(4)]

$$\frac{1}{e} - e\left[1-\frac{\alpha}{4}-(\frac{\alpha}{4})^2 - x(\frac{\alpha}{4})^3\right] = 3 \tag{24}$$

when $x = 0.430 \pm 0.365$.

For example,

$$\begin{aligned} &\text{if}\ \ x = 0.75\ ,\ \ \text{then}\ \alpha^{-1} = 137.035999039\\ &\text{if}\ \ x = 0.50\ ,\ \ \text{then}\ \alpha^{-1} = 137.035999074\\ &\text{if}\ \ x = 0.25\ ,\ \ \text{then}\ \alpha^{-1} = 137.035999109\end{aligned}$$

Comparing Eqs (15) and (24), we get

$$\tan^2(\frac{\pi}{4}-\theta_W) = e^2\left[1-\frac{\alpha}{4}-(\frac{\alpha}{4})^2 - x(\frac{\alpha}{4})^3\right] \tag{25}$$

(In Ref. (4), the following approximation of Eq. (25) is used : $\tan^2(\frac{\pi}{4}-\theta_W) = e^2$).

It is worthwhile to note that

$$\frac{1}{e} - e\left[1-\frac{\alpha}{4}\exp(\frac{\alpha}{4})\right] = 3$$

is satisfied when

$$\alpha^{-1} = 137.035999074.$$

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